## Archimedes' Principle, Buoyancy, and Density

## Equipment

- Chemical splash goggles (Students bring their own)
- Distilled/Deionized Water, Isopropyl alcohol
- Computer with a spreadsheet software
- Set of Digital Calipers
- Force Sensor
- Plastic bins to catch overflow.
- Graduated cylinder
- Aluminum Container with and without spout
- Vertical stand, perpendicular clamp, horizontal rod (between 20 cm and 50 cm )
- Metal Ball with a string attached to it
- Wooden cylinder with pencil lines marking off equal lengths


## Objectives

- Verify Archimedes' principle and use it to determine the density of a given liquid.


## Introduction

The famous legend tells us that Archimedes was the person who discovered that the volume of displaced water equals the volume of a submerged object. He came up with that idea as he was trying to measure the volume of a crown of unusual shape. Puzzled, he had filled his bathtub flush with water and water overflowed when he got inside of the tub. The idea that the amount of water splashed out of the tub is exactly the volume of his own body struck him and he ran outside of his house crying "Eureka!" This means, "I have found it".

Archimedes' Principle itself isn't directly about volume, it's about buoyancy. It states that the buoyant upward force acting on an object entirely or partially submerged in a fluid is equal to the weight of the fluid displaced by the object.

For a given object, the weight can be directly calculated from the mass or from the density and volume:

$$
F_{g}=m g=\rho V g
$$

The buoyant force is found by applying the same idea to the fluid instead of the object:

$$
\begin{equation*}
F_{B}=m_{\text {fluid }} g=\rho_{\text {fluid }} V_{\text {displaced }} g \tag{1}
\end{equation*}
$$

Here, $m_{\text {fluid }}$ is the mass of the displaced fluid, which is broken down as the density of the fluid $\rho_{\text {fluid }}$ multiplied by the submerged volume of the object $V_{\text {displaced }}$.

For a prism-shaped object like a cylinder, the submerged volume is equal to the cross-sectional area, $A$, multiplied by the submerged depth, $d$. So, the buoyant force can be written as:

$$
\begin{equation*}
F_{B}=\rho_{\text {fluid }} A d g \tag{2}
\end{equation*}
$$

If the object is lowered into the fluid while the buoyant force is measured, the slope of the graph of $F_{B}$ versus $d$ is proportional to the density of the fluid.

## Part 1. Volume of the Displaced Liquid

The purpose of this experiment is to verify Archimedes' "finding" that the volume of the displaced liquid is the same as the volume of the object immersed. A metal ball will be used as the solid object.

1. Find the volume of the ball by measuring its diameter and using that to calculate the volume of the ball, assuming that it is a perfect sphere.
2. Submerge the ball in water and determine the volume of the water displaced. (See Figure 1.)

- Place the aluminum container (the one with a spout) in position where you can catch any overflow with the graduated cylinder.
- Fill the container with water so it just overflows (don't catch this water), then allow it to stop dripping. Note that if you move the container after it's full, you'll slosh some water out, so get the container fixed in position first, then fill it.
- Prepare to catch any additional water that comes out of the spout with the graduated cylinder.
- Lower the ball in the water while catching the overflow in the graduated cylinder.


Figure 1. Ready to perform Part 1. The upper container (with the spout) was set in place, and then it was "topped off" with water. (The extra water fell into the plastic bin.) The graduated cylinder is ready to catch the overflow that will come out when the brass ball is lowered into the water. (Do not copy our picture into your lab report!)
3. Measure the displaced water volume using the graduated cylinder. Keep this cylinder of water for Experiment 2!
4. Compare the results. If you're far off, try it again, being more careful not to lose water before submersing the ball. (Note: $1 \mathrm{~cm}^{3}=1 \mathrm{ml}$ )

| Ball diameter (cm) |  |
| :---: | :--- |
| $\mathrm{V}_{\text {ball }}\left(\mathrm{cm}^{3}\right)$ |  |
| $\mathrm{V}_{\text {water }}(\mathrm{ml})$ |  |
| \% Difference |  |

Table 1: Replace this text with an appropriate caption.

## Part 2. Weight of the Displaced Liquid

The purpose of this experiment is to verify Archimedes' Principle for a submerged object. This principle states that the buoyant force acting on a submerged object equals the weight of displaced liquid.

1. Open Capstone File "Force Sensor - Digits Display".
2. Suspend the ball from the hook of the Force Sensor and record its weight in the air.
3. Bring a cup of water up from underneath the ball. (Figure 3)
4. Make sure the ball isn't touching the side or bottom of the cup. While the ball is submerged, record the weight of the ball in water.
5. Using one of the digital balances, find the mass of the graduated cylinder from Experiment 1 with and without the water.
6. Calculate the buoyancy force as the difference of the weight of the ball in air and in water.
7. Calculate the mass of the displaced water as the difference in mass of the graduated cylinder with and without water. Convert the final value to kilograms.
8. Calculate the weight of the displaced water.
9. Compare the experimental value of the buoyancy force with the weight of the displaced water by calculating \% Difference $=$ ABS(Value $1-$ Value 2)/AVERAGE(Value 1,Value 2).

| $W_{\text {in air }}$ <br> $(\mathrm{N})$ | $W_{\text {in water }}$ <br> $(\mathrm{N})$ | $F_{B}$ <br> $(\mathrm{~N})$ | $m_{\text {without water }}$ <br> $(\mathrm{g})$ | $m_{\text {with water }}$ <br> $(\mathrm{g})$ | $m_{\text {water }}$ <br> $(\mathrm{kg})$ | $F_{g, \text { water }}$ <br> $(\mathrm{N})$ | $\%$ <br> Difference |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |

Table 2: Replace this text with an appropriate caption.

## Part 3. Density of a Liquid

The purpose of this experiment is to apply Archimedes' principle to determine the density of a given liquid. For objects with constant cross-sectional areas $(A)$ such as cylinders or cubes one can use Equation 2 above.

From this equation, you can see that the magnitude of the buoyant force is directly proportional to the portion of the object submerged in the liquid $d$. The graph $F_{B}$ vs. $d$ is a linear function with slope equal to $\rho_{\text {water }} A g$.


Figure 2. Taking data for Part 3. The object is lowered into a cup of water until it reached the desired line. The force recorded by the sensor is less than the weight of the block in the air because the buoyancy force supports the object.

1. Suspend the wooden block from the Force Sensor. This tension value could be called $F_{T, a i r}$, but it is the same as $F_{g}$, so record the value and copy it down the $F_{g}$ column.
2. Submerge the wooden block suspended from the force sensor down in water to the first pencil mark ( 1 cm depth). Record tension while the block is in water as $F_{T \text {,water }}$.

- Either lower the block into the water or raise the water under the block.

3. Measure $F_{T, \text { water }}$ for each depth mark until the wooden cylinder starts floating.
4. Once the block starts floating, or if the block leans against the side of the container, don't record any more data.
5. Record your experimental data in a table similar to Table 3.
6. Calculate the buoyant force ( $F_{B}=F_{g}-F_{T, \text { water }}$ ) for each data point.
7. Plot the buoyant force $F_{B}$ versus the submerged depth, $d$.
8. Determine the slope of the graph, measure the area of the base of the block, and compute the density of the liquid. Record this in something similar to Table 3 a.
9. Calculate the \% Error of your density computed from the graph slope, comparing it to the accepted value (google it and give a reference).
10. Repeat 2-9 but using isopropyl alcohol as the fluid instead of water. Label data tables for this part as 4 and 4a. Have a similar approach to graph labeling.

| Depth $\boldsymbol{d}$ (m) | $F_{g}(\mathbf{N})$ | $F_{T, \text { water }}(\mathbf{N})$ | $F_{\boldsymbol{B}}(\mathbf{N})$ |
| :--- | :--- | :--- | :--- |
| 0.01 |  |  |  |
| 0.02 |  |  |  |
| 0.03 |  |  |  |
| $\ldots$ |  |  |  |
|  |  |  |  |
|  |  |  |  |

Table 3: Replace this text with an appropriate caption.

| Slope of $F_{B}$ vs. $d(\mathrm{~N} / \mathrm{m})$ |  |
| :---: | :--- |
| Area of base of block $\left(\mathrm{m}^{\wedge} 2\right)$ |  |
| Density of liquid, $\rho\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ |  |
| Expected $\rho\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ |  |
| $\%$ Error |  |

Table 3a. Replace this text with an appropriate caption.

- Wear chemical safety goggles when alcohol is being used. If you get isopropyl alcohol in your eye, go to the eyewash station and flush it out.
- Don't drink isopropyl alcohol; it's poisonous and not drinkable.
- If you get isopropyl alcohol on your skin, dry it off. Isopropyl alcohol is also known as rubbing alcohol and getting a small amount on your skin shouldn't harm you.
- Don't spill alcohol.
- Ask the instructor about waste collection.


## Requirements for the Report (also consult the rubric):

Save your Excel files through your Blackboard Group File Exchange.

- The abstract section must contain the following explanations in paragraph form:
- The purpose of the experiment: what did you try to verify and why? Make sure to define buoyancy and its relationship with Archimedes Principle.
- Data collection and calculations for Table 1: what tool was used to measure the diameter of the ball and how this measurement was used for the calculation of the volume of the ball; how the displaced water was collected, and the volume of the displaced water was calculated.
- Data collection and calculation for Table 2: what tool was used to measure the weight of the ball in the air/water and how the buoyant force was computed from these measured values; explain what was measured to compute the weight of the displaced water and how these measurements were done.
- Data collection and calculation for Tables 3 and 3 a including a comment on how the manipulation of the submerged depth was reflected in the measured values of the buoyant force.
- How the data from Tables 3 and 4 was analyzed including interpretation of the trendlines (how the slope of the graph was used to calculate the value of the density).
- Summary of the results: does the experiment support Archimedes Principle? Use values from tables 1 and 2 to report on what was expected and if it was achieved. Use values from tables 3a and 4a to discuss if the Archimedes Principle was useful in the calculation of the densities (\% Error calculated for the experimental and accepted values of the densities).
- The data section must include:
- 6 Tables (labeled and captioned)
- 2 Graphs or 1 Graph with 2 data series (titled, axis labels, units, labeled and captioned)

